E3 review key

Stat 422

(1) 8.6

Summary statistics

	n	mean	sum	stdev
m	25	27.12	678	7.4516
y_i	25	1398.3	34957	394.52
$y_i - \bar{y}m_i$	25	0	0	103.97

$N = 108$

By Equation (8.1), the estimate of the population mean μ is,

$$\bar{y} = \frac{\sum y_i}{\sum m_i} = \frac{34957}{678} = 51.56$$

Since M is not known, the $\overline{M}$ appearing in Equation (8.2) must be estimated by $\overline{m}$, where

$$\overline{m} = \frac{\sum m_i}{n} = \frac{678}{25} = 27.12$$

The from Equation (8.2), the estimated variance of population mean is,

$$\hat{V}(\bar{y}) = \left(\frac{N-n}{Nn\overline{M}^2}\right)\frac{1}{n-1}\sum(y_i - \bar{y}m_i)^2 = \left(\frac{N-n}{Nn\overline{M}^2}\right)s_r^2$$

$$= \left(\frac{108-25}{108(25)(27.12)^2}\right)(103.97)^2$$

with

$$B = 2\sqrt{\hat{V}(\bar{y})} = 1.344$$

(2) 8.18

$N = 48, \; n = 10$

$\sum y_i = 736$

$\sum m_i = 365$

$$\bar{y}_t = \frac{\sum y_i}{n} = \frac{736}{10} = 73.6$$

$$\hat{\tau} = N\bar{y}_t = 48(73.6) = 3532.8$$

$$\hat{V}(\hat{\tau}) = N^2\left(\frac{N-n}{Nn}\right)s_t^2$$

$$= 48^2\left(\frac{48-10}{48(10)}\right)(398.93)$$

$$B = 2\sqrt{\hat{V}(\hat{\tau})} = 539.50$$

(3) 8.21

a_i = number of defective microchips on board i
m_i = number of microchips on board i (12 per board)

$n = 10$, $\overline{M} = 12$

$$\sum a_i = 16$$

$$\sum m_i = 120$$

$$\hat{p} = \frac{\sum a_i}{\sum m_i} = \frac{16}{120} = .1333$$

By Equation (8.17), ignoring the fpc,

$$\hat{V}(\hat{p}) = \left(\frac{1}{n\overline{M}^2}\right) s_p^2$$

$$= \frac{1}{10(12^2)}(2.046)$$

$$B = 2\sqrt{\hat{V}(\hat{p})} = .075$$

Plot	M_i	m_i	$\bar{y}_i$	s_i	$M_i\bar{y}_i$	$M_i(\bar{y}_i - \hat{\mu}_r)$	*within*
1	52	5	11.6000	1.14018	603.200	115.496	635.440
2	56	6	8.8333	1.16905	494.667	−30.553	637.778
3	60	6	5.5000	1.04881	330.000	−232.776	594.000
4	46	5	7.0000	0.70711	322.000	−109.431	188.600
5	49	5	11.6000	1.14018	568.400	108.832	560.560
6	51	5	13.4000	1.14108	683.400	205.075	609.960
7	50	5	6.8000	0.83666	340.000	−128.947	315.000
8	61	6	9.1667	0.75277	559.167	−12.948	316.861
9	60	6	8.8333	1.16905	530.000	−32.736	738.000
10	45	6	12.0000	0.63246	540.000	117.948	117.000

where *within* is defined as $M_i(M_i - m_i)s_i^2 / m_i$

Summary statistics

	n	mean	stdev
M_i	10	53.00	5.907
m_i	10	5.50	.5271
$M_i\bar{y}_i$	10	497.1	125.21
$M_i(\bar{y}_i - \hat{\mu}_r)$	10	0	135.85
within	10	471.32	216.48

Since M is unknown, we must use $\hat{\mu}_r$, given by Equation (9.7), to estimate μ.

$$\hat{\mu}_r = \frac{\sum M_i\bar{y}_i}{\sum M_i} = \frac{\sum M_i\bar{y}_i / n}{\sum M_i / n} = \frac{497.1}{53} = 9.379$$

The estimated variance of $\hat{\mu}_r$ is, from Equation (9.8),

Figure 1: 9.2 part 1

(4) 9.2

$$\hat{V}(\hat{\mu}_r) = \frac{N-n}{Nn\overline{M}^2} s_r^2 + \frac{1}{Nn\overline{M}^2} \sum M_i^2 \frac{M_i - m_i}{M_i} \frac{s_i^2}{m_i}$$

$$= \frac{50-10}{50(10)(53)^2}(135.85)^2 + \frac{1}{50(10)(53)^2}(4713.2)$$

($\overline{M}$ is estimated by $\overline{m}$)

$$B = 2\sqrt{\hat{V}(\hat{\mu}_r)} = 1.455$$

where, from above summary statistics table,

$$\overline{m} = 53\,, \quad s_r^2 = \frac{1}{n-1}\sum M_i^2 (\overline{y}_i - \hat{\mu}_r)^2 = (135.85)^2$$

$$\sum M_i^2 \frac{M_i - m_i}{M_i} \frac{s_i^2}{m_i} = \sum M_i \left(M_i - m_i \right) \frac{s_i^2}{m_i} = 4713.2$$

$N = 7, \quad n = 3$

Area	M_i	m_i	$\hat{p}_i$	$M_i \hat{p}_i$	$M_i(\hat{p}_i - \hat{p})$	*within*
1	46	9	0.111111	5.1111	−0.40862	21.0123
2	67	13	0.153846	10.3077	2.26808	39.2485
3	93	20	0.100000	9.3000	−1.85946	32.1584

where *within* is defined as $M_i(M_i - m_i)\hat{p}_i(1-\hat{p}_i)/(m_i - 1)$

Figure 2: 9.7 part 1

(6) 9.7

Summary statistics

	n	mean	stdev
M_i	3	68.6667	23.544
m_i	3	14.0	5.5678
$M_i \hat{p}_i$	3	8.2396	2.7558
$M_i(\hat{p}_i - \hat{p})$	3	0	2.0939
within	3	30.806	9.1830

$$\hat{p} = \frac{\sum M_i \hat{p}_i}{\sum M_i} = \frac{\sum M_i \hat{p}_i / n}{\sum M_i / n} = \frac{8.2396}{68.6667} = .120$$

$$\hat{V}(\hat{p}) = \left(\frac{N-n}{N}\right)\frac{1}{n\bar{M}^2}s_r^2 + \frac{1}{nN\bar{M}^2}\sum M_i^2\left(\frac{M_i - m_i}{M_i}\right)\left(\frac{\hat{p}_i\hat{q}_i}{m_i - 1}\right)$$

$$= \left(\frac{7-3}{7}\right)\frac{1}{3(68.6667)^2}(2.0939)^2 + \frac{1}{3(7)(68.6667)^2}(92.418)$$

$$B = 2\sqrt{\hat{V}(\hat{p})} = .067$$

where

$$s_r^2 = \frac{1}{n-1}\sum M_i^2(\hat{p}_i - \hat{p})^2 = (2.0939)^2$$

$$\sum M_i^2\left(\frac{M_i - m_i}{M_i}\right)\left(\frac{\hat{p}_i\hat{q}_i}{m_i - 1}\right) = 30.806 \times 3 = 92.418$$

(7) 10.5

$n = 515, \quad t = 320, \quad s = 91$

$$\hat{N} = \frac{nt}{s} = \frac{515(320)}{91} = 1810.99 \approx 1811$$

$$\hat{V}(\hat{N}) = \frac{t^2 n(n-s)}{s^3} = \frac{(320)^2(515)(515-91)}{(91)^3}$$

$$B = 2\sqrt{\hat{V}(\hat{N})} = 344.51$$

(8) 10.11

$n = 75,\quad t = 100,\quad s = 10$

$$\hat{N} = \frac{nt}{s} = \frac{75(100)}{10} = 750$$

$$\hat{V}(\hat{N}) = \frac{t^2 n(n-s)}{s^3} = \frac{(100)^2(75)(75-10)}{(10)^3}$$

$$B = 2\sqrt{\hat{V}(\hat{N})} = 441.59$$

(9) 10.16

$a = 10$ minutes
$A = 8$ hours $\times$ 60 minutes/hour = 480 minutes
$\overline{m} = 40, \; n = 20$

Using Equation (10.6) , we determine the estimated density as

$$\hat{\lambda} = \frac{\overline{m}}{a} = \frac{40}{10} = 4 \text{ cars per min.}$$

Using Equation (10.8), the estimated total number pf cars for a eight-hour period is,

$$\hat{M} = \hat{\lambda}A = 4(480) = 1920 \text{ cars}$$

with a bound on the error, from Equation (10.9), of

$$B = 2A\sqrt{\frac{\hat{\lambda}}{an}} = 2(480)\sqrt{\frac{4}{10(20)}} = 135.76$$

$a = 1$ field
$n = 240$ fields

$$\sum m_i = 0(11) + 1(37) + 2(64) + 3(65) + 4(37) + 5(24) + 6(12) = 670$$

$$\overline{m} = \frac{\sum m_i}{n} = \frac{670}{240} = 2.792$$

$$\hat{\lambda} = \frac{\overline{m}}{a} = \frac{2.79}{1} = 2.792$$

$$B = 2\sqrt{\frac{\hat{\lambda}}{an}} = 2\sqrt{\frac{2.792}{(1)(240)}} = .216$$

Figure 3: 10.19

(10) 10.19